Period 1, March 5, 2025

Area of a Square = s^2

Area of a Triangle = $\frac{1}{2}bh$

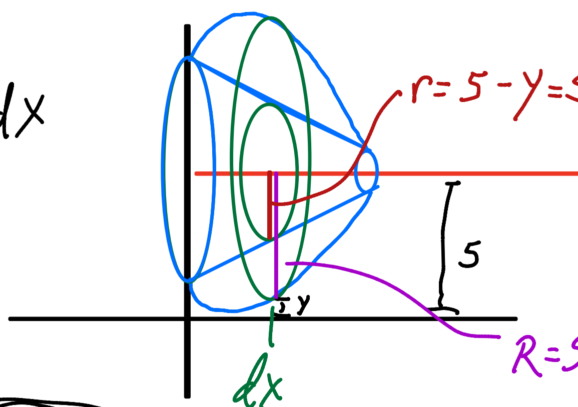
Area of an Equilateral Triangle = $\frac{\sqrt{3}}{4}s^2$

Area of a Semi Circle = $\frac{\pi r^2}{2}$

6.

Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x + 1$ and $y = x^2 - 2x + 1$ about the line $y = 5$.

$$\pi \int_0^3 (R^2 - r^2)\, dx$$



$r = 5 - y = 5 - (x+1) = 4 - x$

$R = 5 - y = R = 5 - (x^2 - 2x + 1)$

$= 4 - x^2 + 2x$

$$\pi \int_0^3 \left[(4 - x^2 + 2x)(4 - x^2 + 2x) - (4 - x)(4 - x)\right] dx$$

$$\pi \int_0^3 \left[16 - \cancel{4x^2} + 8x - \cancel{4x^2} + x^4 - \cancel{2x^3} + 8x - \cancel{2x^3} + \cancel{4x^2} - 16 + 4x + 4x - \cancel{x^2}\right] dx$$

$$\pi \int_0^3 (x^4 - 4x^3 - 5x^2 + 24x)\, dx = \pi \left(\frac{1}{5}x^5 - x^4 - \frac{5}{3}x^3 + 12x^2\right)\Big|_0^3$$

$\frac{1}{5}(3)^5 - 3^4 - \frac{5}{3}(3)^3 + 12(3)^2 - \left[\cancel{\frac{1}{5}(0)^5 - 0^4 - \frac{5}{3}0^3 + 12(0)^2}\right] = \frac{243}{5} - 81 - 45 + 108$

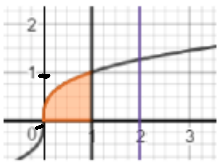
$48\frac{3}{5} - 126 + 108$

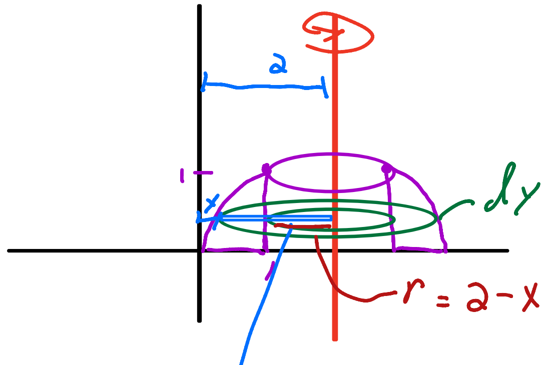
$156\frac{3}{5} - 126 = 30\frac{3}{5} = \frac{153\pi}{5}$

7.

Find the volume of the solid generated by revolving the region bounded by $y = \sqrt[3]{x}$, the x-axis, and the line $x = 1$ about the line $x = 2$.

$y^3 = x$





$r = 2 - x = 2 - 1 = 1$

$x = 2$

$R = 2 - x = 2 - y^3$

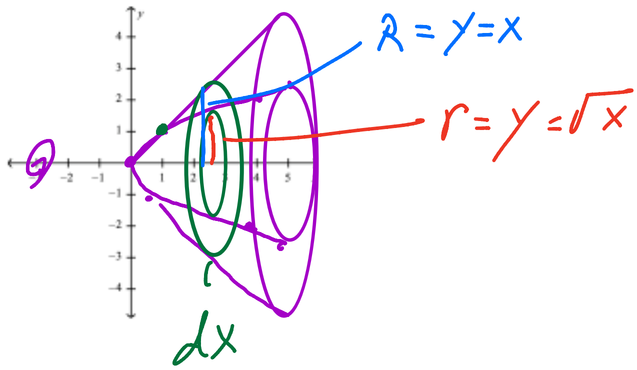
$$\pi\int_0^1 \left[(2-y^3)^2 - 1^2\right] dy$$

$$\pi\int_0^1 \left[4 - 4y^3 + y^6 - 1\right] dy$$

$$\pi\int_0^1 \left[3 - 4y^3 + y^6\right] dy = \pi\left[3y - y^4 + \frac{1}{7}y^7\right]\Big|_0^1$$

$$\pi\left[\left(3\cdot 1 - 1^4 + \frac{1}{7}(1)^7\right) - \left(\cancel{3(0) - 0^4 + \frac{1}{7}\cdot 0^7}\right)\right] = \pi\left(3 - 1 + \frac{1}{7}\right) = 2\frac{1}{7}\pi = \frac{15}{7}\pi$$

2. Find the volume of the solid that results when the region enclosed by the curves $y = \sqrt{x}$, and $x = y$, **and $x = 5$** is revolved about $y = 0$. **[No Calc; solve and simplify this question by hand]**



$$\pi \int_1^5 \left[(x)^2 - (\sqrt{x})^2 \right] dx$$

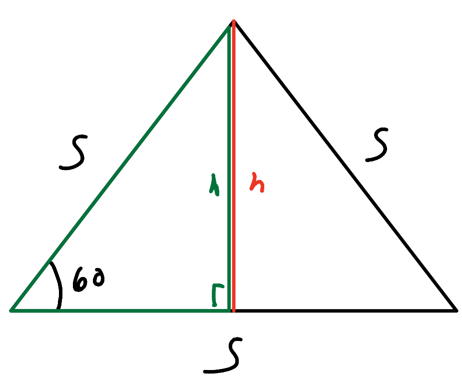
$$\pi \int_1^5 \left[x^2 - x \right] dx$$

$$\pi \left(\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 \right) \Big|_1^5$$

$$\pi \left[\left(\tfrac{1}{3}(5)^3 - \tfrac{1}{2}(5)^2 \right) - \left(\tfrac{1}{3}(1)^3 - \tfrac{1}{2}(1)^2 \right) \right]$$

$$\frac{125}{3} - \frac{25}{2} - \frac{1}{3} + \frac{1}{2}$$

$$\frac{124}{3} - \frac{24}{2} = 41\tfrac{1}{3} - 12 = 29\tfrac{1}{3} \cdot \pi$$



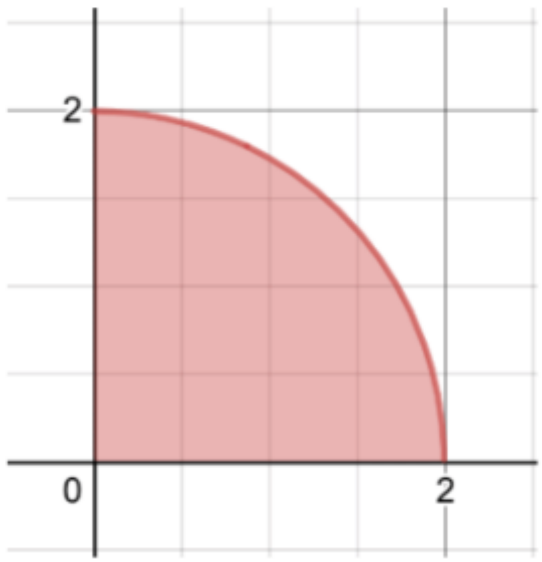
$$\text{Area} = \frac{1}{2} \cdot b \cdot h$$

$$\sin 60 = \frac{h}{s}$$

$$s \cdot \sin 60 = h$$

$$s \cdot \frac{\sqrt{3}}{2} = h$$

$$\text{Area} = \frac{1}{2} \cdot s \cdot s \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} s^2$$



Example 1) Find the volume of the solid whose base is bounded by $x^2 + y^2 = 4$ in the first quadrant, the cross sections perpendicular to the x-axis are squares.

$x^2 + y^2 = 4$

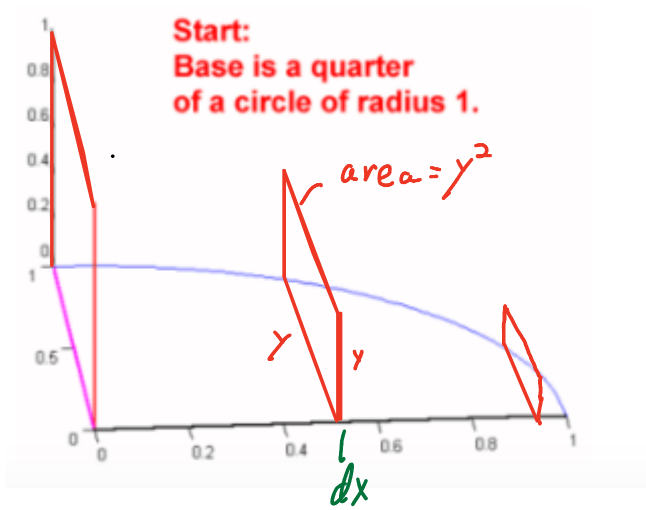
$y^2 = 4 - x^2$

$\int_0^1 y^2\,dx = \int_0^1 (4 - x^2)\,dx$

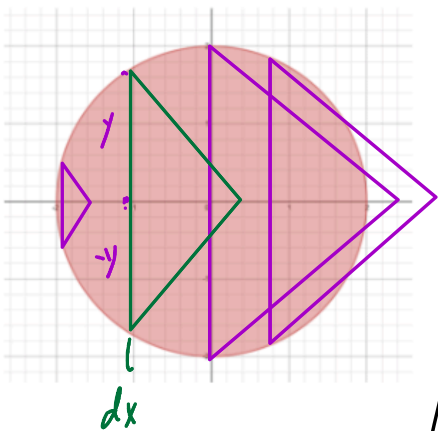
$4x - \frac{1}{3}x^3 \Big]_0^1$

$4(1) - \frac{1}{3}(1)^3 - \left[\cancel{4\cdot 0 - \frac{1}{3}\cdot 0^3}\right]$

$4 - \frac{1}{3} = 3\frac{2}{3}$



Set up the integral that finds the volume of the solid whose base is bounded by $x^2 + y^2 = 4$, the cross sections perpendicular to the x-axis are Equilateral Triangles.



$(\ \frac{\sqrt{3}}{4} \cdot s^2$

$s = 2y$

$\frac{\sqrt{3}}{4} \cdot (2y)^2 = Area$

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$$\int_{-2}^{2} \frac{\sqrt{3}}{4} (2y)^2 \cdot dx$$

$$\int_{-2}^{2} \frac{\sqrt{3}}{\cancel{4}} \cdot \cancel{4} y^2 dx = \sqrt{3} \int_{-2}^{2} y^2 dx = \sqrt{3} \int_{-2}^{2} (4 - x^2) dx$$

$$\sqrt{3} \left[4x - \frac{1}{3} x^3 \right] \Big|_{-2}^{2}$$

$$\sqrt{3} \left[4(2) - \frac{1}{3}(+2)^3 \right] - \sqrt{3} \left[\overset{-8}{4(-2)} - \overset{+\frac{8}{3}}{\frac{1}{3}(-2)^3} \right]$$

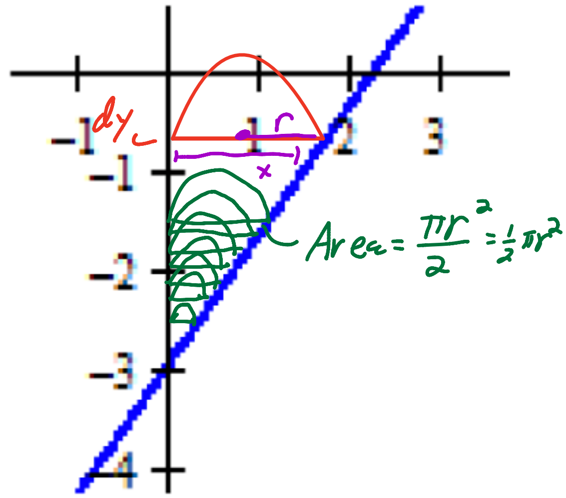
$$8\sqrt{3} - \frac{8\sqrt{3}}{3} + 8\sqrt{3} - \frac{8\sqrt{3}}{3}$$

$$3 \cdot \frac{16\sqrt{3}}{3 \cdot 1} - \frac{16\sqrt{3}}{3} = \frac{(48 - 16)\sqrt{3}}{3} = \frac{32\sqrt{3}}{3}$$

Example 2)

Find the volume of the function $y = \frac{4}{3}x - 3$ bounded by the 4th quadrant if the cross sections perpendicular to the y-axis are semicircles.

$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = \frac{1}{8}\pi x^2$



$\int_{-3}^{0} \frac{1}{8}\pi x^2 \cdot dy$

$y = \frac{4}{3}x - 3$

$\frac{3}{4}(y+3) = \frac{3}{4}\cdot\frac{4}{3}x$

$\frac{3}{4}(y+3) = x$

$\int_{-3}^{0} \frac{1}{8}\pi\left(\frac{3}{4}(y+3)\right)^2 dy = \int_{-3}^{0} \frac{\pi}{8}\cdot\frac{9}{16}(y+3)^2 dy$

$\frac{9\pi}{128}\int_{-3}^{0}(y^2+6y+9)\,dy$

$\frac{9\pi}{128}\left[\frac{1}{3}y^3+3y^2+9y\right]\Big[_{-3}^{0} = \cancel{\frac{9\pi}{128}\left[\frac{1}{3}(0)^3+3(0)^2+9(0)\right]} - \frac{9\pi}{128}\left[\frac{1}{3}(-3)^3+3(-3)^2+9(-3)\right]$

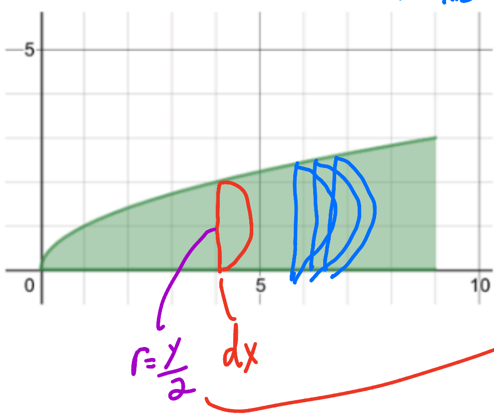
$-\frac{9\pi}{128}\cdot\frac{1}{3}\cdot(-3)^3 - \frac{9\pi}{128}\cdot 3(-3)^2 - \frac{9\pi}{128}\cdot 9(-3)$

$\frac{9\pi}{128}\cdot\frac{1}{3}\cdot 27 - \cancel{\frac{9\pi}{128}\cdot 27} + \cancel{\frac{9\pi}{128}\cdot 27}$

$\frac{9\pi}{128}\cdot 9 = \frac{81\pi}{128}$

Set up the integral that Finds the volume of the function $y = \sqrt{x}$ bounded by the 1st quadrant and $x = 9$ if the cross sections perpendicular to the ~~y-axis~~ are semicircles. $= \frac{\pi r^2}{2}$

x-axis



$$\int_0^9 \frac{\pi r^2}{2}\, dx = \int_0^9 \frac{\pi}{2}\left(\frac{y}{2}\right)^2 dx$$

$$= \int_0^9 \frac{\pi}{2}\cdot\left(\frac{\sqrt{x}}{2}\right)^2 dx$$

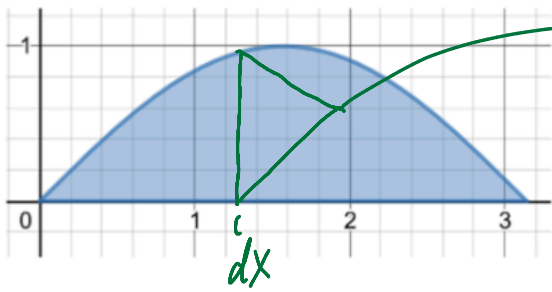
$$\int_0^9 \frac{\pi}{2}\cdot\frac{x}{4}\, dx = \frac{\pi}{8}\int_0^9 x\, dx$$

$$\frac{\pi}{8}\cdot\frac{1}{2}x^2\Big|_0^9 = \frac{\pi}{16}(9)^2 - \frac{\pi}{16}(0)^2$$

Keep going

Example 4)

Find the volume of the function $y = \sin x$ $(0 \le x \le \frac{\pi}{2})$ if the cross sections perpendicular to the x-axis are equilateral Triangles.



$$\text{Area} = \frac{\sqrt{3}}{4} \cdot s^2 = \frac{\sqrt{3}}{4}(y)^2$$

$$\int_0^{\pi/2} \frac{\sqrt{3}}{4} \cdot y^2 \, dx$$

$$\int_0^{\pi/2} \frac{\sqrt{3}}{4} \cdot \sin^2 x \, dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x - 1 = -2\sin^2 x$$

$$\frac{\cos 2x - 1}{-2} = \sin^2 x$$

$$\int_0^{\pi/2} \frac{\sqrt{3}}{4} \cdot \frac{(\cos 2x - 1)}{-2} \, dx$$

$$-\frac{\sqrt{3}}{8} \int_0^{\pi/2} (\cos 2x - 1) \, dx$$

$$-\frac{\sqrt{3}}{8} \int_0^{\pi/2} \cos 2x \, dx - -\frac{\sqrt{3}}{8} \int_0^{\pi/2} 1 \, dx$$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$$-\frac{\sqrt{3}}{8} \int \cos u \cdot \frac{du}{2}$$

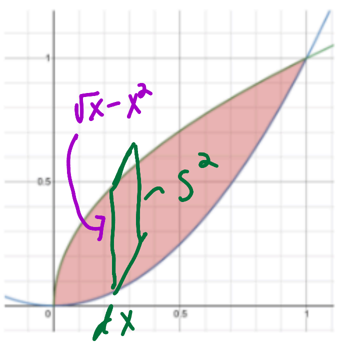
$$-\frac{\sqrt{3}}{16} \sin 2x + \frac{\sqrt{3}}{8} x \Big|_0^{\pi/2}$$

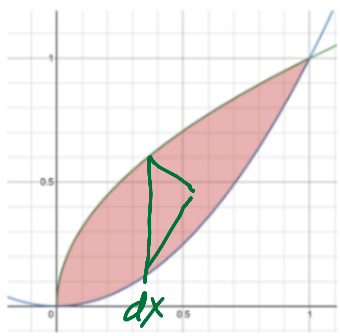
keep going

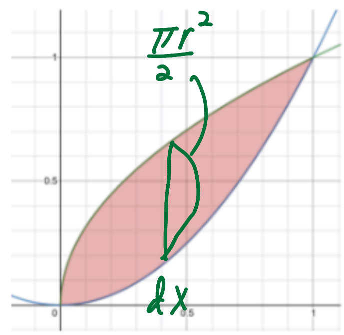
Find the volume of the solid whose base is bounded by $y = \sqrt{x}$ and $y = x^2$, the cross sections perpendicular to the x-axis are a) Squares b) Eq. Tri c) Semi Circles

$A = \frac{\sqrt{3}}{4} \cdot S^2$

https://www.geogebra.org/m/XFgMaKTy







$\int_0^1 (\sqrt{x} - x^2)^2 \, dx =$

$\int_0^1 \frac{\sqrt{3}}{4} (\sqrt{x} - x^2)^2 \, dx =$

$\int_0^1 \frac{1}{2}\pi \left(\frac{\sqrt{x} - x^2}{2}\right)^2 dx$

$(\sqrt{x} - x^2)(\sqrt{x} - x^2) = x - x^{5/2} - x^{5/2} + x^4 = x - 2x^{5/2} + x^4$

$\int_0^1 (x - 2x^{5/2} + x^4) \, dx$

$\frac{1}{2}x^2 - 2 \cdot \frac{2}{7} x^{\frac{5}{2}+1=\frac{7}{2}} + \frac{1}{5}x^5$

$\frac{1}{2}x^2 - \frac{4}{7}x^{\frac{7}{2}} + \frac{1}{5}x^5 \Big|_0^1$

$\frac{\sqrt{3}}{4} \int_0^1 (x - 2x^{5/2} + x^4) \, dx$

$\frac{\sqrt{3}}{4} \left(\frac{1}{2}x^2 - \frac{4}{7}x^{7/2} + \frac{1}{5}x^5\right) \Big|_0^1$

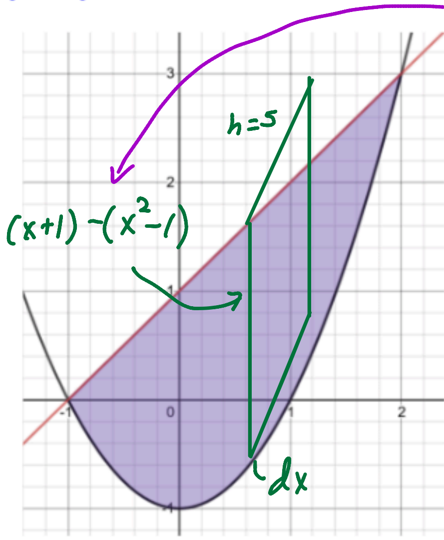
$\frac{\pi}{2} \int_0^1 \frac{(x - 2x^{5/2} + x^4)}{4} \, dx$

$\frac{\pi}{8} \int_0^1 (x - 2x^{5/2} + x^4) \, dx$

$\frac{\pi}{8} \left(\frac{1}{2}x^2 - \frac{4}{7}x^{\frac{7}{2}} + \frac{1}{5}x^5\right) \Big|_0^1$

Example 6)

Find the volume of the solid whose base is bounded by $y = x + 1$ and $y = x^2 - 1$, the cross sections perpendicular to the x-axis are rectangles of height 5.



$$\int_{-1}^{2} [(x+1) - (x^2-1)] \cdot 5 \cdot dx$$

$$\int_{-1}^{2} [x+1 - x^2 + 1] \cdot 5\, dx$$

$$5 \int_{-1}^{2} [-x^2 + x + 2]\, dx$$

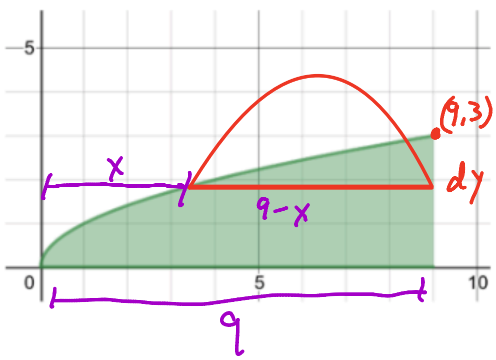
$$5\left[-\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + 2x\right]\Big|_{-1}^{2}$$

$$5\left[-\tfrac{1}{3}(2)^3 + \tfrac{1}{2}(2)^2 + 2\cdot 2\right] - 5\left[-\tfrac{1}{3}(-1)^3 + \tfrac{1}{2}(-1)^2 + 2(-1)\right]$$

$$-\frac{40}{3} + 10 + 20 - \frac{5}{3} - \frac{5}{2} + 10 = -\frac{45}{3} + 30 - 2\tfrac{1}{2} + 10 = \underbrace{-15 + 40}_{25 - 2\frac{1}{2}} - 2\tfrac{1}{2}$$

$$22\tfrac{1}{2}$$

Set up the integral that Finds the volume of the function $y = \sqrt{x}$ bounded by the 1st quadrant and $x = 9$ if the cross sections perpendicular to the y-axis are semicircles.



$y^2 = x$

$r = \frac{9-x}{2}$

$$\int_0^3 \frac{\pi r^2}{2}\, dy = \frac{\pi}{2}\int_0^3 \left(\frac{9-x}{2}\right)^2 dy$$

$$\frac{\pi}{2}\int_0^3 \left(\frac{9-y^2}{2}\right)^2 dy$$

keep going